LEY DE NEWTON DE LA VISCOSIDAD

PARA FLUJO INCOMPRESIBLE

$$\boldsymbol{\tau} = -\mu\dot{\boldsymbol{\gamma}}$$

Coordenadas Rectangulares

esfuerzos normales	esfuerzos tangenciales
$\tau_{xx} = -2\mu \frac{\partial v_x}{\partial x}$	$\tau_{xy} = \tau_{yx} = -\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$
$\tau_{yy} = -2\mu \frac{\partial v_y}{\partial y}$	$\tau_{xz} = \tau_{zx} = -\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$
$\tau_{zz} = -2\mu \frac{\partial v_z}{\partial z}$	$\tau_{yz} = \tau_{zy} = -\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$

Coordenadas Cilíndricas

esfuerzos normales	esfuerzos tangenciales
$\tau_{rr} = -2\mu \frac{\partial v_r}{\partial r}$	$\tau_{r\theta} = \tau_{\theta r} = -\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$
$\tau_{\theta\theta} = -2\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)$	$\tau_{rz} = \tau_{zr} = -\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$
$\tau_{zz} = -2\mu \frac{\partial v_z}{\partial z}$	$\tau_{\theta z} = \tau_{z\theta} = -\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$

Coordenadas Esféricas

esfuerzos normales	esfuerzos tangenciales
$\tau_{rr} = -2\mu \frac{\partial v_r}{\partial r}$	$\tau_{r\theta} = \tau_{\theta r} = -\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$
$\tau_{\theta\theta} = -2\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)$	$\tau_{r\phi} = \tau_{\phi r} = -\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_r}{\partial \phi}\right)$
$\tau_{\phi\phi} = -2\mu\left(\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right)$	$\tau_{\theta\phi} = \tau_{\phi\theta} = -\mu\left(\frac{\mathrm{sen}\,\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\mathrm{sen}\,\theta}\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\theta}{\partial \phi}\right)$

NOTACIÓN:

$\boldsymbol{\tau}$ = tensor simétrico del esfuerzo cortante (Pa)

μ = viscosidad (Pa·s = kg/m·s)

$\mathbf{v}$ = vector de velocidad (m/s)

$\dot{\boldsymbol{\gamma}} = \left[(\nabla\mathbf{v}) + (\nabla\mathbf{v})^{\mathsf{T}}\right]$ = tensor simétrico de rapidez de deformación (s^{-1})

$\nabla\mathbf{v}$ = tensor del gradiente de velocidad (s^{-1})

$(\nabla\mathbf{v})^{\mathsf{T}}$ = transpuesta del tensor del gradiente de velocidad (s^{-1})

GRADIENTE DE VELOCIDAD ($\nabla \mathbf{v}$) RAPIDEZ DE DEFORMACIÓN ($\dot{\gamma}$)

Coordenadas Rectangulares

$$\nabla \mathbf{v} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_y}{\partial x} & \frac{\partial v_z}{\partial x} \\ \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} & \frac{\partial v_z}{\partial y} \\ \frac{\partial v_x}{\partial z} & \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

$$\dot{\boldsymbol{\gamma}} = \begin{bmatrix} 2\frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \\ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} & 2\frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \\ \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} & \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} & 2\frac{\partial v_z}{\partial z} \end{bmatrix}$$

Coordenadas Cilíndricas

$$\nabla \mathbf{v} = \begin{bmatrix} \frac{\partial v_r}{\partial r} & \frac{\partial v_\theta}{\partial r} & \frac{\partial v_z}{\partial r} \\ \frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} & \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} & \frac{1}{r}\frac{\partial v_z}{\partial \theta} \\ \frac{\partial v_r}{\partial z} & \frac{\partial v_\theta}{\partial z} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

$$\dot{\boldsymbol{\gamma}} = \begin{bmatrix} 2\frac{\partial v_r}{\partial r} & r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} & \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \\ r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} & \frac{2}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right) & \frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \\ \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} & \frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} & 2\frac{\partial v_z}{\partial z} \end{bmatrix}$$

Coordenadas Esféricas

$$\nabla \mathbf{v} = \begin{bmatrix} \frac{\partial v_r}{\partial r} & \frac{\partial v_\theta}{\partial r} & \frac{\partial v_\phi}{\partial r} \\ \frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} & \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} & \frac{1}{r}\frac{\partial v_\phi}{\partial \theta} \\ \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\phi}{r} & \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\theta}{\partial \phi} - \frac{v_\phi}{r}\cot\theta & \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\phi}{r}\cot\theta \end{bmatrix}$$

$$\dot{\boldsymbol{\gamma}} = \begin{bmatrix} 2\frac{\partial v_r}{\partial r} & r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} & r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_r}{\partial \phi} \\ r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} & \frac{2}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right) & \frac{\mathrm{sen}\,\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\mathrm{sen}\,\theta}\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\theta}{\partial \phi} \\ r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_r}{\partial \phi} & \frac{\mathrm{sen}\,\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\mathrm{sen}\,\theta}\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\theta}{\partial \phi} & \frac{2}{r}\left(\frac{1}{\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} + v_r + v_\theta\cot\theta\right) \end{bmatrix}$$

NOTACIÓN:

$\mathbf{v}$ = vector de velocidad (m/s)

$\dot{\boldsymbol{\gamma}} = \left[(\nabla\mathbf{v}) + (\nabla\mathbf{v})^{\mathsf{T}}\right]$ = tensor simétrico de rapidez de deformación (s^{-1})

$\nabla\mathbf{v}$ = tensor del gradiente de velocidad (s^{-1})

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