OPERACIONES QUE INVOLUCRAN ∇ EN LOS DIFERENTES SISTEMAS COORDENADOS

TABLA A.7−1

RESUMEN DE OPERACIONES DIFERENCIALES EN LAS QUE INTERVIENE EL OPERADOR ∇ EN COORDENADAS RECTANGULARES[a] (x, y, z)

$$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \qquad (A)$$

$$(\nabla^2 s) = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} \qquad (B)$$

$$(\boldsymbol{\tau} : \nabla \mathbf{v}) = \tau_{xx}\left(\frac{\partial v_x}{\partial x}\right) + \tau_{yy}\left(\frac{\partial v_y}{\partial y}\right) + \tau_{zz}\left(\frac{\partial v_z}{\partial z}\right) + \tau_{xy}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) + \tau_{yz}\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right) + \tau_{zx}\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right) \qquad (C)$$

$$[\nabla s]_x = \frac{\partial s}{\partial x} \qquad (D) \qquad\qquad [\nabla \times \mathbf{v}]_x = \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \qquad (G)$$

$$[\nabla s]_y = \frac{\partial s}{\partial y} \qquad (E) \qquad\qquad [\nabla \times \mathbf{v}]_y = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \qquad (H)$$

$$[\nabla s]_z = \frac{\partial s}{\partial z} \qquad (F) \qquad\qquad [\nabla \times \mathbf{v}]_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \qquad (I)$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \qquad (J)$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \qquad (K)$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \qquad (L)$$

$$[\nabla^2 \mathbf{v}]_x = \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \qquad (M)$$

$$[\nabla^2 \mathbf{v}]_y = \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \qquad (N)$$

$$[\nabla^2 \mathbf{v}]_z = \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \qquad (O)$$

$$[\mathbf{v} \cdot \nabla \mathbf{v}]_x = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \qquad (P)$$

$$[\mathbf{v} \cdot \nabla \mathbf{v}]_y = v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \qquad (Q)$$

$$[\mathbf{v} \cdot \nabla \mathbf{v}]_z = v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \qquad (R)$$

[a] Las operaciones en las que interviene el tensor $\boldsymbol{\tau}$ están dadas **solamente para** $\boldsymbol{\tau}$ **simétrico**

TABLA A.7−2

RESUMEN DE OPERACIONES DIFERENCIALES EN LAS QUE INTERVIENE EL OPERADOR ∇ EN COORDENADAS CILÍNDRICAS[a] (x, θ, z)

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \qquad (A)$$

$$(\nabla^2 s) = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2} \qquad (B)$$

$$(\boldsymbol{\tau} : \nabla \mathbf{v}) = \tau_{rr}\left(\frac{\partial v_r}{\partial r}\right) + \tau_{\theta\theta}\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \tau_{zz}\left(\frac{\partial v_z}{\partial z}\right) + \tau_{r\theta}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] + \tau_{\theta z}\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right) + \tau_{rz}\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right) \qquad (C)$$

$$[\nabla s]_r = \frac{\partial s}{\partial r} \qquad (D) \qquad\qquad [\nabla \times \mathbf{v}]_r = \frac{1}{r}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z} \qquad (G)$$

$$[\nabla s]_\theta = \frac{1}{r}\frac{\partial s}{\partial \theta} \qquad (E) \qquad\qquad [\nabla \times \mathbf{v}]_\theta = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \qquad (H)$$

$$[\nabla s]_z = \frac{\partial s}{\partial z} \qquad (F) \qquad\qquad [\nabla \times \mathbf{v}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r v_\theta) - \frac{1}{r}\frac{\partial v_r}{\partial \theta} \qquad (I)$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{r\theta} - \frac{1}{r}\tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z} \qquad (J)$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r}\tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z} \qquad (K)$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} \qquad (L)$$

$$[\nabla^2 \mathbf{v}]_r = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \qquad (M)$$

$$[\nabla^2 \mathbf{v}]_\theta = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \qquad (N)$$

$$[\nabla^2 \mathbf{v}]_z = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \qquad (O)$$

$$[\mathbf{v} \cdot \nabla \mathbf{v}]_r = v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \qquad (P)$$

$$[\mathbf{v} \cdot \nabla \mathbf{v}]_\theta = v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \qquad (Q)$$

$$[\mathbf{v} \cdot \nabla \mathbf{v}]_z = v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \qquad (R)$$

[a] Las operaciones en las que interviene el tensor $\boldsymbol{\tau}$ están dadas **solamente para** $\boldsymbol{\tau}$ **simétrico**

Adaptado de: Bird, Steward y Lightfoot, "Fenómenos de Transporte", Ed. Reverté.

OPERACIONES QUE INVOLUCRAN ∇ EN LOS DIFERENTES SISTEMAS COORDENADOS

TABLA A.7−3

RESUMEN DE OPERACIONES DIFERENCIALES EN LAS QUE INTERVIENE EL OPERADOR ∇ EN COORDENADAS ESFÉRICAS[a] (r, θ, ϕ)

$$(\nabla \cdot \boldsymbol{v}) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial}{\partial \theta}(v_\theta\,\text{sen}\,\theta) + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} \qquad (A)$$

$$(\nabla^2 s) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\,\text{sen}\,\theta}\frac{\partial}{\partial \theta}\left(\text{sen}\,\theta\frac{\partial s}{\partial \theta}\right) + \frac{1}{r^2\text{sen}^2\,\theta}\frac{\partial^2 s}{\partial \phi^2} \qquad (B)$$

$$\begin{aligned}(\boldsymbol{\tau} : \nabla \boldsymbol{v}) = {} & \tau_{rr}\left(\frac{\partial v_r}{\partial r}\right) + \tau_{\theta\theta}\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) \\ & + \tau_{\phi\phi}\left(\frac{1}{r\,\text{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right) \\ & + \tau_{r\theta}\left(\frac{\partial v_\theta}{\partial r} + \frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r}\right) + \tau_{r\phi}\left(\frac{\partial v_\phi}{\partial r} + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\phi}{r}\right) \\ & + \tau_{\theta\phi}\left(\frac{1}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial v_\theta}{\partial \phi} - \frac{\cot\theta}{r}v_\phi\right) \qquad (C)\end{aligned}$$

$$[\nabla s]_r = \frac{\partial s}{\partial r} \qquad (D)$$

$$[\nabla s]_\theta = \frac{1}{r}\frac{\partial s}{\partial \theta} \qquad (E)$$

$$[\nabla s]_\phi = \frac{1}{r\,\text{sen}\,\theta}\frac{\partial s}{\partial \phi} \qquad (F)$$

$$[\nabla \times \boldsymbol{v}]_r = \frac{1}{r\,\text{sen}\,\theta}\frac{\partial}{\partial \theta}(v_\phi\,\text{sen}\,\theta) - \frac{1}{r\,\text{sen}\,\theta}\frac{\partial v_\theta}{\partial \phi} \qquad (G)$$

$$[\nabla \times \boldsymbol{v}]_\theta = \frac{1}{r\,\text{sen}\,\theta}\frac{\partial v_r}{\partial \phi} - \frac{1}{r}\frac{\partial}{\partial r}(r v_\phi) \qquad (H)$$

$$[\nabla \times \boldsymbol{v}]_\phi = \frac{1}{r}\frac{\partial}{\partial r}(r v_\theta) - \frac{1}{r}\frac{\partial v_r}{\partial \theta} \qquad (I)$$

[a]Las operaciones en las que interviene el tensor $\boldsymbol{\tau}$ están dadas **solamente para $\boldsymbol{\tau}$ simétrico**

TABLA A.7−3 (*continuación*)

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\,\text{sen}\,\theta) + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r} \qquad (J)$$

$$\begin{aligned}[\nabla \cdot \boldsymbol{\tau}]_\theta = {} & \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\,\text{sen}\,\theta) + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} \\ & + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi} \qquad (K)\end{aligned}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\phi}) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\phi}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi} \qquad (L)$$

$$[\nabla^2 \boldsymbol{v}]_r = \nabla^2 v_r - \frac{2v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} - \frac{2v_\theta\cot\theta}{r^2} - \frac{2}{r^2\text{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} \qquad (M)$$

$$[\nabla^2 \boldsymbol{v}]_\theta = \nabla^2 v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2\text{sen}^2\,\theta} - \frac{2\cos\theta}{r^2\text{sen}^2\,\theta}\frac{\partial v_\phi}{\partial \phi} \qquad (N)$$

$$[\nabla^2 \boldsymbol{v}]_\phi = \nabla^2 v_\phi - \frac{v_\phi}{r^2\text{sen}^2\,\theta} + \frac{2}{r^2\,\text{sen}\,\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cos\theta}{r^2\text{sen}^2\,\theta}\frac{\partial v_\theta}{\partial \phi} \qquad (O)$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\,\text{sen}\,\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \qquad (P)$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\,\text{sen}\,\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r} \qquad (Q)$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\,\text{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi\cot\theta}{r} \qquad (R)$$

Adaptado de: Bird, Steward y Lightfoot, "Fenómenos de Transporte", Ed. Reverté.